

The density profiles of Dark Matter halos in Spiral Galaxies

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ABSTRACT

In spiral galaxies, we explain their non-Keplerian rotation curves (RCs) by means of a non-luminous component embedding their stellar-gaseous disks. Understanding the detailed properties of this component (labelled Dark Matter, DM) is one of the most pressing issues of Cosmology. We investigate the recent relationship (claimed by Walker et al. 2010, hereafter W+10) between r , the galaxy radial coordinate, and $V_h(r)$, the dark halo contribution to the circular velocity at r , *a)* in the framework of the Universal Rotation Curve (URC) paradigm and directly *b)* by means of the kinematics of a large sample of DM dominated spirals. We find a general agreement between the W+10 claim, the distribution of DM emerging from the URC and that inferred in the (low luminosity) objects of our sample. We show that such a phenomenology, linking the spiral's luminosity, radii and circular velocities, implies an evident inconsistency with (naive) predictions in the Λ Cold Dark Matter (Λ CDM) scenario.

Keywords: galaxies: spiral, kinematics and dynamics; dark matter

I. INTRODUCTION

Rotation curves (RCs) of spiral galaxies show no asymptotic Keplerian behavior and fail to match the distribution of the luminous matter. The favored explanation is the existence of an unknown unseen component, the Dark Matter (DM): the “luminous” components of galaxies such as gas and stars are distributed inside a spherical “dark” halo ¹.

Recently, [2], hereafter W+10, following the pioneer paper of [3] (M+07), found that the dark component of the circular velocity (and of mass) profile shows a characteristic and similar behavior both in Spirals (and in Milky Way dSphs) of very different luminosities. More precisely, they investigated a sample of 60 spirals, whose

RCs span radii from 1 kpc to 75 kpc, have maximum amplitudes in the range $50 \text{ km s}^{-1} \leq V_{max} \leq 300 \text{ km s}^{-1}$ and their baryonic masses M_b ranging between $3 \times 10^8 M_\odot$ and $4 \times 10^{11} M_\odot$. They derived $V_h(r)$, the halo contributions to the circular velocity (in short the “halo RCs”), by subtracting the contribution of the baryonic matter from the circular velocity: $V_h^2(r) = V^2(r) - V_b^2(r)$ (see W+10 for details). They found a correlation between $V_h(r)$ and r (see W+10):

$$\log \left(\frac{V_h(r)}{\text{km s}^{-1}} \right) = 1.47_{-0.19}^{+0.15} + 0.5 \log \left(\frac{r}{\text{kpc}} \right), \quad (1)$$

independently of galaxy luminosity. Moreover: $\frac{M_h(r)}{M_\odot} = 2_{-1.2}^{+2} \times 10^8 \left(\frac{r}{\text{kpc}} \right)^2$. Noticeably, at $r = R_{1/2}$, being $R_{1/2}$ the radius encompassing half of the *stellar* mass, the relation (1) can be investigated also in dwarf spheroidals, and found to hold also in these systems (see W+10 for details).

These are surprising findings: in fact, when, in objects of *very different* luminosity we derive their $V_h(r)$ (the *dark halo-contribution* to the circular velocities) and plot them as a function of radius, a well defined curve emerges, instead of a more likely scatter plot. On the other hand, the *circular velocities* $V(r)$ show a very different phenomenology. From the analysis of several thousands of RCs, an *Universal Rotation Curve* (URC) emerges, i.e. a specific 3-dimensional surface linking *a)* the circular velocity at a *normalized* radius r/R_D (i.e. measured in units of disk length-scale), *b)* the galaxy luminosity and *c)*, the normalized radius (see [4], [5] (PSS), [6], [7] and [8] (S+07) for details).

Summarizing, in the intermediate regions of spirals, W+10 found: $V_h(r) = F_{W+10}(r)$, independently of galaxy luminosity L and stellar disk length-scale R_D . In the URC scenario, instead, we have: $V_h(r) = F_{URC}(r/R_D, L)$, i.e. the halo component of the circular velocity is a function of disk luminosity and disk length scale. Let us stress that the existence of the URC implies that the structural parameters of the Dark and Luminous components are strongly related, but not necessarily, through the very constraining eq. (1).

In this paper, we investigate whether these two apparently different results are compatible. We also use the outcome to derive important information on the DM distribution in spirals. We will study an independent (larger) sample of spirals by means of reliable method of mass modelling. From [9] (PS95) we select 116 spiral galaxies (hereafter Sample A) whose RCs satisfy the following requirements: *a)* no presence of a prominent

¹ A thorough review on this issue can be found in [1] and downloaded at www.sissa.it/ap/dmg/dmaw-presentation.html.

bulge: $V(0.2R_{opt}) < 50 \text{ km s}^{-1}$, or if the latter is missing: $V(0.4R_{opt}) < 70 \text{ km s}^{-1}$, b) low luminosity objects: $V(r) < 120 \text{ km s}^{-1}$, at any radius, that implies: $M_I > -20.2$, with M_I the magnitude in the I band, c) the RCs have a measure either at $0.8 R_{opt}$ or at $1.0 R_{opt}$. Let us notice that we limit ourselves to objects of low luminosity, in that they are DM dominated (e.g. [11] (PS90)) and therefore $V_h(r)$ can be derived with great accuracy. Higher luminosity spirals are disk dominated and therefore $V_h(r)$ strongly depends on the details and on the assumptions in the mass modelling. The RCs of Sample A have a radial range matching that of W+10 sample $\max[\frac{1}{3}R_D, 1\text{kpc}] \leq r \leq 6R_D$. Following W+10, we adopted 1 kpc as the innermost radius in that, inside which, the stellar disk (sometimes with a stellar bulge) dominates the gravitational potential making difficult the estimate of $V_h(r)$.

Let us compare the W+10 relationship with the URC analogue (PSS, S+07) and with that we derive for Sample A. From the condition of centrifugal equilibrium we have:

$$V^2(r) = V_{lum}^2(r) + V_h^2(r), \quad (2)$$

where the subscripts *lum* and *h* stand respectively for luminous and halo components. The former is the quadratic sum of disk, bulge and gas contributions. Here, we neglect the latter two because 1) the selected low luminosity objects have, for $r > 1 \text{ kpc}$, a negligible bulge, 2) the gaseous disk is important only for $r > R_{opt}$, i.e. outside the region that we are considering here (see [12]). For the objects of Sample A, each RC has 2-4 independent measurements (see Table 2 of PS95) and therefore the same number of halo velocity data. For the URC, there is analytical formula for $V_h(r)$ (see below). Hereafter, we adopt a flat cosmology with matter density parameter $\Omega_M = 0.27$ and Hubble constant $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

II. THE CONTRIBUTION OF DARK MATTER HALOS TO THE CIRCULAR VELOCITY OF SPIRALS

We derive the velocity halo contribution for the objects of Sample A in the following way; we assume a Freeman profile to describe the stellar disk surface density, then [10]:

$$V_D^2(x) = 14.9 \beta(x) V^2(1.0) x^2 [I_0 K_0 - I_1 K_1]_{1.6x}, \quad (3)$$

where $x = r/R_{opt}$, I_n and K_n are the modified Bessel functions. Let us also notice that: $R_{1/2} =$

$1.67/3.2 R_{opt} = 0.52 R_{opt}$. The mass modelling of individual (and coadded) RCs of objects of low luminosity (as those in Sample A) is rather simple. We have: $\beta(1) \simeq 0.13$ and $\beta(0.8) \simeq 0.23$ (see PS90 and PSS). More precisely, there is a small luminosity dependence of β but for the objects in Sample A this is irrelevant. Then, for any object, we derive $V_h(r)$ by subtracting the (small) disk contribution $V_D^2(r)$, given by eq. (3), from the circular velocity $V^2(r)$. Finally, we derive $V_h(R_{1/2})$ by linearly interpolating $V_h(0.4R_{opt})$ and $V_h(0.6R_{opt})$. The URC leads to an (analytical) Universal form for $V_h(r)$, i.e. for the *halo* velocity component (URCH) to $V_{URC}(r)$ which was built by coadding the kinematics of thousands of galaxies (PSS, S+07). We model the URC, that represents the typical RC of an object of magnitude M_I , (or of virial halo mass M_{vir} (see S+07 and reference therein) in its dark and luminous components (e.g. S+07)³). In detail the URC DM density profile is

$$\rho_{URCH}(r) = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)}, \quad (4)$$

where r_0 is the core radius and ρ_0 is the central halo density. It follows that:

$$V_{URCH}^2(r) = 6.4 \frac{\rho_0 r_0^3}{r} \left[\ln \left(1 + \frac{r}{r_0} \right) - \arctan \left(\frac{r}{r_0} \right) + \frac{1}{2} \ln \left(1 + \frac{r^2}{r_0^2} \right) \right]. \quad (5)$$

From [14] we have:

$$\rho_0 = 5 \times 10^{-24} (r_0 / (8.6 \text{ kpc}))^{-1} \text{ g cm}^{-3}, \quad (6)$$

and

$$\log \left(\frac{r_0}{\text{kpc}} \right) \simeq 0.66 + 0.58 \log \left(\frac{M_{vir}}{10^{11} M_\odot} \right). \quad (7)$$

The disk length-scales and masses, R_D and M_D , are related to the halo masses through (see [15] and S+07)

$$\log \left(\frac{R_D}{\text{kpc}} \right) = 0.633 + 0.379 \log \left(\frac{M_D}{10^{11} M_\odot} \right) + 0.069 \left(\log \frac{M_D}{10^{11} M_\odot} \right)^2, \quad (8)$$

and

$$M_D = 2.3 \times 10^{10} M_\odot \frac{[M_{vir}/(3 \times 10^{11} M_\odot)]^{3.1}}{1 + [M_{vir}/(3 \times 10^{11} M_\odot)]^{2.2}}. \quad (9)$$

By means of equations (4)-(9) we can derive, the URCH halo velocity $V_h^{URC}(r/R_D, L)$.

² R_{opt} is the radius encircling 83% of the light, for a Freeman exponential disk $R_{opt} = 3.2R_D$ where R_D is the disk scale-length, see [10].

³ The virial mass M_{vir} corresponds to the mass enclosed in a sphere with density $101 \rho_c$, being ρ_c the critical density of the Universe. M_{vir} and the virial radius are related by $R_{vir} = 259 (M_{vir}/10^{12} M_\odot)^{1/3} \text{ kpc}$, see [13].

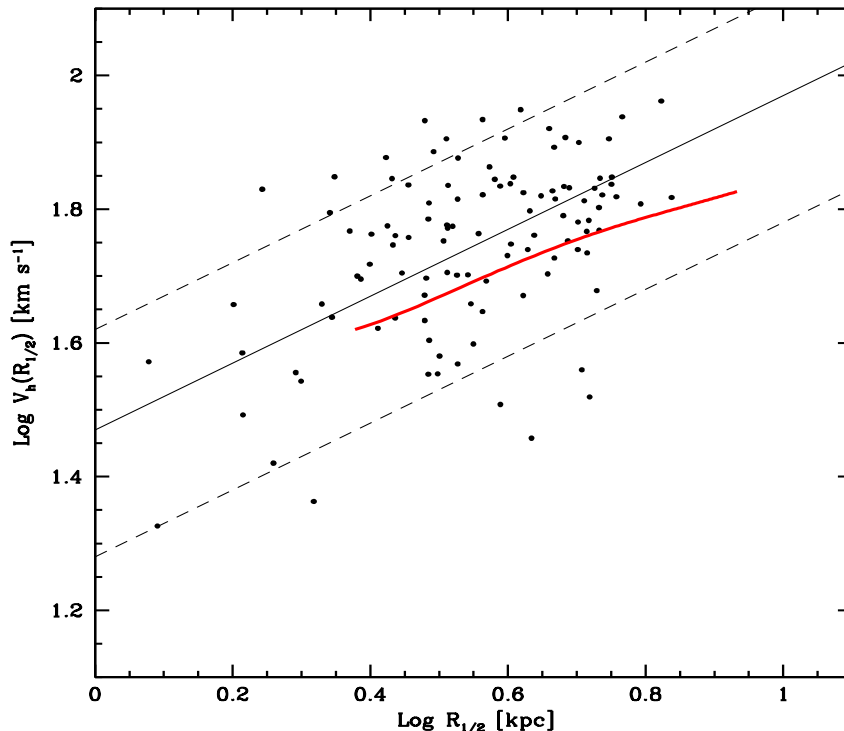


FIG. 1: The W+10 halo velocity-radius relationship (black line) with its $1\text{-}\sigma$ scatter (black dashed lines) compared with the halo velocity calculated at $R_{1/2}$ as function of $R_{1/2}$ for 1) our Sample A whose points are calculated by interpolating $V_h(0.4R_{opt})$ and $V_h(0.6R_{opt})$ for each galaxy (black points) and 2) URCH profiles (red line).

The first step is to investigate the $V_h(R_{1/2}) - R_{1/2}$ relationship, that holds also for objects of different Hubble types. In Fig. (1) we plot the URCH as a red line and the relation for Sample A as black points. It is clear that both individual and URC halo velocities do correlate with $R_{1/2}$, well matching the W+10 (M+07) relation (shown as black line).

The further step is to investigate the full profile $V_h(r)$ at “intermediate radii” $\max[\frac{1}{3}R_D, 1\text{kpc}] \leq r \leq 6R_D$. Each of the 116 spirals contributes with 2-4 independent kinematical measurements. In Fig. (2) we plot $V_h(r)$ obtained from the RCs of Sample A, once that the luminous component has been removed as explained above. We also plot, as two red lines, the URCH velocity profiles of spirals with luminosities similar to those of the objects of Sample A ⁴, and to the great majority of the 60 spirals of the W+10 sample. Notice that, given the

Schechter-like form of the luminosity function of spirals, in the W+10 sample there are only a handful of big galaxies.

In the low luminosity range we plot $V_h(r)$ for the individual RCs of Sample A (black points) and for the corresponding URCH profiles, and compare them with the W+10 relationship. Let us recall that we did not investigate the *individual* $V_h(r)$ of luminous spirals since their dark-luminous RC decomposition is quite uncertain. However, we have investigated the high luminosity objects by means of the URCH, that we show in Fig. (3) alongside with the W+10 relationship. Summarizing, for spirals in halos with $M_{vir} > 1.9 \times 10^{12} M_\odot$, we derive $V_h(r)$ by means of the URCH, for halos of lower mass, instead, we derive $V_h(r)$ from 1) the URCH (red lines) and 2) the mass modelling of the individual RCs of Sample A (black points). In both cases, the (derived) halo velocities are compared with the W+10 relationship.

⁴ The related virial masses are $M_{vir}/M_\odot = 1.17 \times 10^{11}, 7.7 \times 10^{11}$.

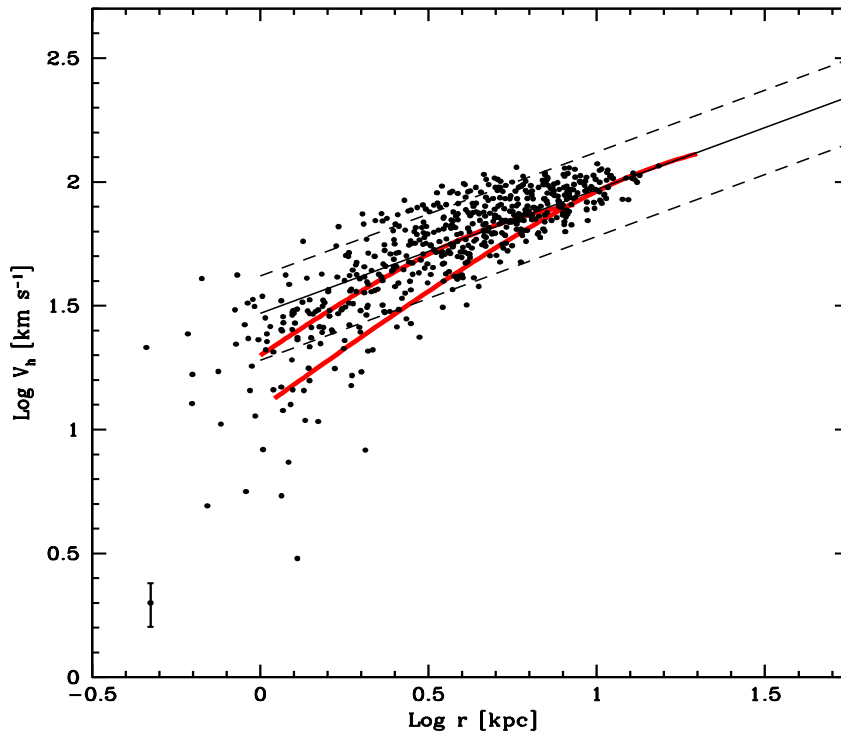


FIG. 2: The W+10 halo velocity-radius relationship (black line) with its $1\text{-}\sigma$ scatter (black dashed lines) plotted with 1) the halo velocities of our Sample A of 116 spirals (black points) (PSS, PS90), 2) the URCH profiles (red lines), corresponding to objects with mass comparable with those in the W+10 sample. The error (of 20 %) in the individual determination of V_h is shown as an errorbar.

III. UNIVERSAL HALO VELOCITY PROFILE AND IMPLICATIONS

We find for Sample A and also from the URC, that: $V_h(R_{1/2}) \propto R_{1/2}^{0.5}$, very similar to the W+10 relation. This result is not a new one, it arises as effect of the particular dependencies of the dark and luminous structural parameters on luminosity and disk length scale. We can show this by looking at the Radial Tully Fisher relation ([16]). From their eq. (9) one has $L_I \propto V(R_{1/2})^{2.55}$, where L_I is the I-band luminosity, considering that for spirals we have: $R_{1/2} \propto R_D \propto L^{0.5}$, and that from the mass modelling of individual objects $V(R_{1/2})/V_h(R_{1/2}) \propto V_h(R_{1/2})^{0.2}$ (see PS90, [17, 18], i.e. smaller galaxies have a larger fraction of DM), one arrives to $V_h(R_{1/2}) \propto R_{1/2}^k$, with k very near to the value of +0.5, claimed by W+10 (see Fig. (1)).

However, with the complete data from the URC shown in Fig. (4) we realize that $V_h(r)$ in Spirals of different luminosities, has not an unique profile, neither always it

is a power law with exponent $1/2$. However, at low luminosities, the halo velocity individual contributions and the URCH profiles are in good agreement with the W+10 result (see also M+07).

At highest luminosities (see Fig. (4)) the W+10 relationship cannot be reproduced by the URCH profiles. However let us stress that in these cases $V_h(r)$ depend significantly (especially in the W+10 work) on the assumptions adopted in the mass modelling. Moreover, the URCH profiles are obtained from hundreds of RCs, while $V_h(r)$ in W+10 only from few (see W+10).

As a result, in a first approximation we can claim that the W+10 relationship, in the radial range $1 \text{ kpc} \leq r \leq 30 \text{ kpc}$, is a projection, on the $(r, V_h(r))$ axes of a more complex $V_{URC}(r/R_{opt}, L)$ relationship in which the structural quantities $V_{URC}(1, L)$, L and R_D are related in a specific way. This suggests the existence of a number of scaling laws between disk scalenghts, disk mass, halo structural parameters and galaxy luminosity, which by the way have been also observed through the analysis of accurate mass models in large samples of galaxies (e.g. S+07).

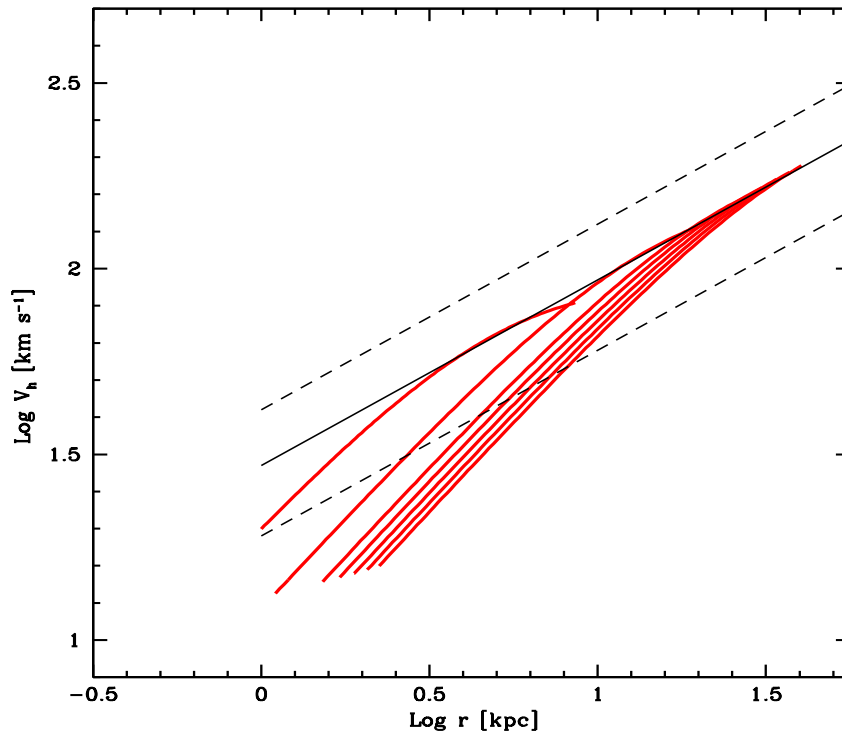


FIG. 3: The W+10 halo velocity-radius relationship (black line) with its $1\text{-}\sigma$ scatter (black dashed lines) plotted with the URCH profiles (red lines).

The W+10 relation URC-supported, becomes very useful to test whether DM halos in spirals are consistent with NFW density profile. In this case let us remind that (see [19]):

$$V_{NFW}^2(r) = V_{vir}^2 \frac{g(c)}{xg(cx)}, \quad (10)$$

where $x = r/R_{vir}$ is the radial coordinate, $g(c) = [\ln(1+c) - c/(1+c)]^{-1}$, and for the concentration parameter c we take

$$c(M_{vir}) = 9.2 \left(\frac{M_{vir}}{10^{12} h^{-1} M_{\odot}} \right)^{-0.09}, \quad (11)$$

(see [1, 20]). The halo mass inside a radius r is given by:

$$M_{NFW}(x) = M_{vir} \frac{g(c)}{g(cx)}. \quad (12)$$

In Fig. (4) we plot $V_{NFW}(r, M_{vir})$ for the radial and halo mass ranges identical with those investigated above (more specifically we consider 8 masses equally log-spaced in the range: $7.7 \times 10^{11} \leq M_{vir}/M_{\odot} \leq 1.1 \times 10^{13}$).

The resulting halo velocity curves $V_{NFW}(r)$ occupies a very well defined portion in the $(r, V_h(r))$ plane (blue lines). However, this region is mostly off that defined by 1) the W+10 relationship, 2) the similar relationship derived in this paper by a larger number of individual RCs (of low luminosity objects) and 3) a specific projection of the URCH (given in Fig. (2)). A disagreement of the NFW halo velocity profiles with the actual kinematics of spirals is not new (e.g. [21]). However, here, we make a step further by investigating an unprecedented large number of low luminosity objects. We derive their $V_h(r)$ profiles from disk kinematics by adopting only weak and reasonable assumptions. These profiles result incompatible with the NFW profiles *for any value of their masses and their concentrations*.

IV. DISCUSSION

We have compared the W+10 ($V_h(R_{1/2}) - R_{1/2}$) relationship (see also M+07), with the URCH and with the kinematics of 116 spirals of low luminosity objects. We

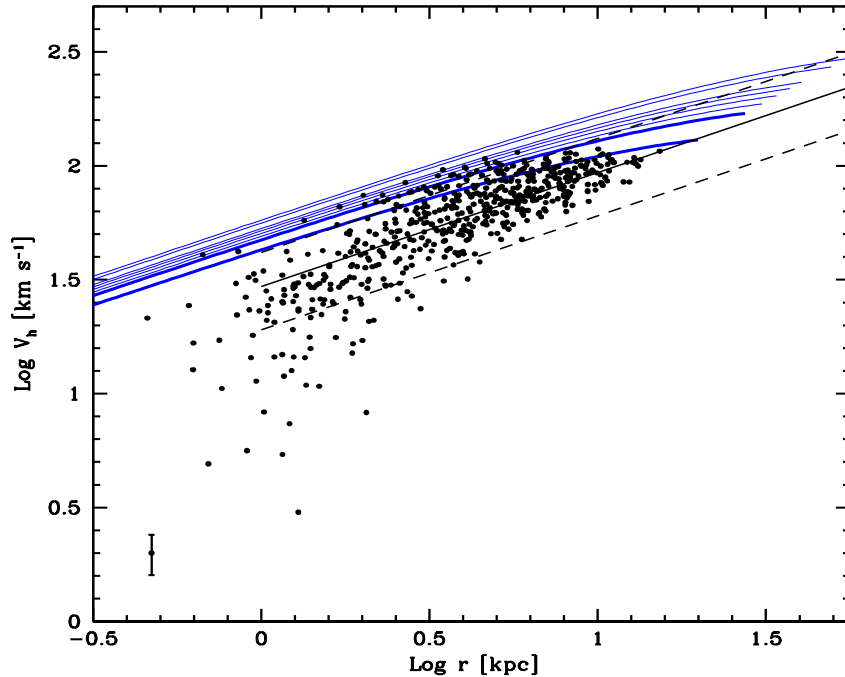


FIG. 4: The W+10 halo velocity-radius relationship (black line) with its $1\text{-}\sigma$ scatter (black dashed lines) plotted with 1) the halo velocities of our Sample A of 116 spirals (black points), 2) the NFW halo velocity profiles (blue lines, with the thick curves corresponding to objects with mass comparable with those of the individual Sample).

find that the W+10 “unique” velocity profile is essentially in agreement with the URC paradigm of which it is a sort of projection. The W+10 relation alongside its not negligible scatter of 0.25 dex, can be identified as the projection of the URC on the $(r, V_h(r))$ plane. This implies the existence of a number of scaling laws between disk scalenghts, disk mass, halo structural parameters and galaxy luminosity, observed also analyzing accurate mass models in large samples of galaxies.

We claim that the distribution of dark and luminous matter in galaxies is such that at any chosen radius r , $V_h(r)$ takes, in different galaxies of different mass, almost the same value. The simplest explanation is that the more massive is the dark halo, the lesser dense and concentrated it turns out to be. The net effect is that the dark mass inside a chosen physical radius is weakly dependent of the total halo mass. The physical explanation of this still lags: we realize that the dark and the luminous matter are distributed in a related way, which is not obvious, in view of their very different nature. It is worth to point out that the URCH profiles for virial masses

$M_{vir} > 3 \times 10^{12} M_\odot$, are discrepant with the W+10 relationship, also considering its (quite large) scatter (see Fig. (3)). Although in the W+10 sample there is an obvious shortage of such big objects, it would seem that for the about 5% top luminous objects, the halo velocity profile implied by W+10 does not agree with the URC mass model and it indicates a larger amount of DM. This issue will be clarified only with a bigger sample of individual RCs. At low masses, low circular velocities and low radii ($1 \text{ kpc} < r < 30 \text{ kpc}$), we find the existence of an “unique” $V_h(r)$ profile whose physical meaning has to be understood. This is amazing in that by adopting the radial units, i.e. kpc, we lose any autosimilarity of the velocity/mass profiles, present when the radial coordinate is measured in terms of R_{vir} or R_D . We can use the W+10 relationship as a Cosmology Test: in fact, when plotted in radial physical units, the NFW velocity profiles lie in a very small region, almost independently of their halo masses and concentration. Such a region, however, turns out to be clearly outside that indicated by the derived halo velocity profiles in W+10, in the URCH

and in our sample of individual RCs. Especially at low luminosities, where the NFW velocities are 0.2-0.4 dex higher, the observed discrepancy is very severe in that, being the contribution of the luminous matter quite negligible, $V_h(r)$ is virtually derived in a model independent way, and it almost coincides with the observed $V(r)$. Of course, the inconsistency between RCs and NFW halo + disk mass model is already well known, even if it is proved

only in a quite limited number of test cases ([22]). Here we have opened the possibility to investigate such a crucial issue in direct way and by means of a large number of objects. It is worth recalling that also within Λ CDM scenario there are several ways to modify the original NFW halo profiles to be compatible with observations, see [23–25] and references therein.

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